Chapter 2: Frequency Distributions and Graphs

Calculation-example mean, median, midrange, mode, variance, and standard deviation for raw and grouped data

Raw data: 7, 8, 6, 3, 5, 5, 1, 6, 4, 10

Sorted data: 1, 3, 4, 5, 5, 6, 6, 7, 8, 10

Number of observations (n) = 10

Sum of the raw data ($\sum x$): 1+3+4+5+5+6+6+7+8+10 = 55

Mean (μ): $(\sum x)/n$ = 55/10 = 5.5

Median (MD): sorted data ~~1~~, ~~3~~, ~~4~~, ~~5~~, 5, 6, ~~6~~, ~~7~~, ~~8~~, ~~10~~ take the (average of the) middle number(s) (5+6)/2 = 5.5

Midrange (MR): sorted data: 1, 3, 4, 5, 5, 6, 6, 7, 8, 10 take the average of the first and the last number (1+10)/2 = 5.5

Mode: the numbers in the raw data that appear the most, 5 and 6

Note: the mean, median, and midrange values are only coincidentally equal in this example

Note: the mean, median, and midrange values do not have to be actual observations that appear in the raw data

Note: the data is bi-modally distributed

Variance (s^2 for sample data or σ^2 for population data): subtract the mean value from every number, square the result and add all these results together, then divide the sum by $n-1$ for the sample-standard deviation and by n for the population-standard deviation

$(1 - 5.5)^2 = 20.25$

$(3 - 5.5)^2 = 6.25$

$(4 - 5.5)^2 = 2.25$

$(5 - 5.5)^2 = 0.25$

$(5 - 5.5)^2 = 0.25$

$(6 - 5.5)^2 = 0.25$

$(6 - 5.5)^2 = 0.25$

$(7 - 5.5)^2 = 2.25$

$(8 - 5.5)^2 = 6.25$

$(10 - 5.5)^2 = 20.25$

20.25+6.25+2.25+0.25+0.25+0.25+0.25+2.25+6.25+20.25 = 58.5

s^2 = 58.5 / (10-1) = 6.5 and σ^2 = 58.5 / 10 = 5.85

Standard deviation (s for sample data or σ for population data): take the square root of the variance $s = \sqrt{6.5}$ = 2.55 and $\sigma = \sqrt{5.85}$ = 2.42

Create a histogram with four classes and calculate the statistics/parameters (mean, median, and mode) from the grouped sample/population data

Range of values: sorted data 1, 3, 4, 5, 5, 6, 6, 7, 8, 10 subtract first number from last number 10-1 = 9

Class-width: divide range of values by number of classes and round up to the same number of decimals as the raw data 9/4 = 2.25 rounded to 3

Determine the first and second class to establish a pattern:

Class 1: first number is 1; class-width is three so in class the numbers 1, 2, and 3

Class 2: first number is 4; class-width is three so in class the numbers 4, 5, and 6

Pattern:

Class 1: 1 – 3

Class 2: 4 – 6

Class 3: 7 – 9

Class 4: 10 – 12

Note: the pattern for the lower class-limits 1, 4, 7, and 10 (add three to previous number); pattern for upper class-limits 3, 6, 9, and 12 (add three to previous number)

Class-limits: class 1: 1 – 3, class 2: 4- 6, class 3: 7 – 9, class 4: 10 – 12

Class-boundaries: class 1: 0.5 – 3.5, class 2: 3.5- 6.5, class 3: 6.5 – 9.5, class 4: 9.5 – 12.5

For classes with decimal class-limits:

Class-limits: class 1: 0.5 – 2.5, class 2: 2.6- 4.6, class 3: 4.7 – 6.7, class 4: 6.8 – 8.8

Class-boundaries: class 1: 0.45 – 2.55, class 2: 2.55- 4.65, class 3: 4.65 – 6.75, class 4: 6.75 – 8.85

Note: the class-limits should have the same decimal place value as the data, but the class-boundaries should have one additional place value and end in a five

Note: the class-width can be found by subtracting the lower boundary from the upper boundary for any given class; do NOT subtract the lower limit from the upper limit of a single class since that will result in an incorrect answer

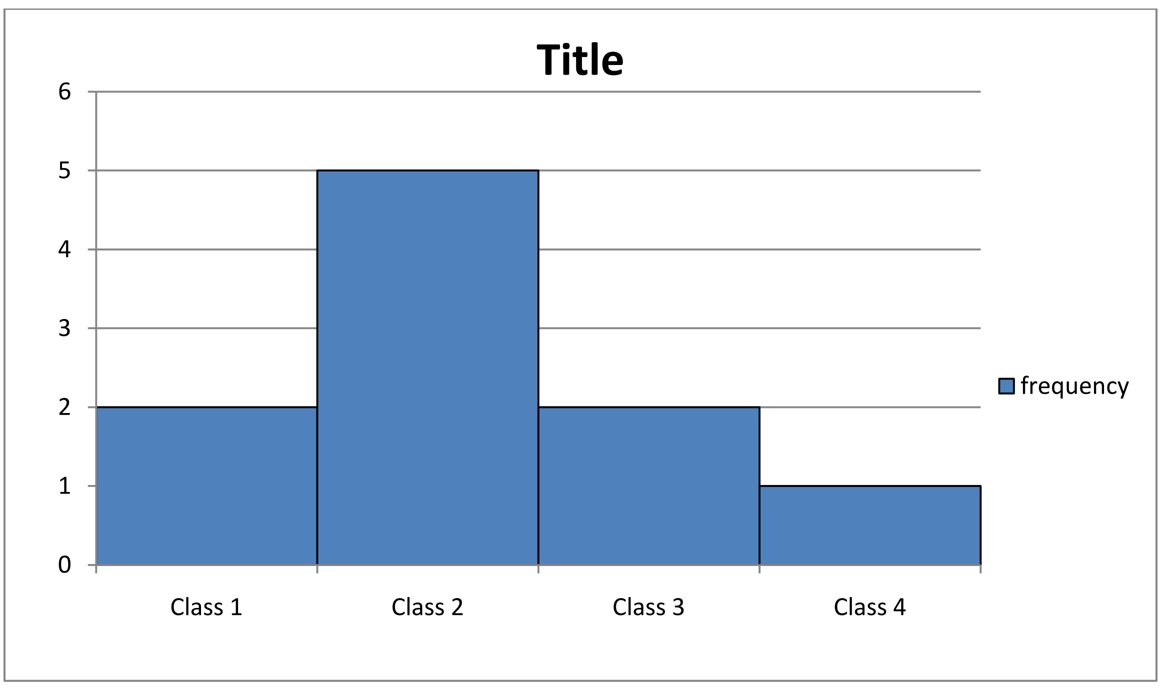
Classes and their frequencies:

Sorted data: 1, 3, 4, 5, 5, 6, 6, 7, 8, 10

Class	Frequency (f)	Cumulative frequency ($\sum f$)
Class 1: 0.5 – 3.5	2	2
Class 2: 3.5 – 6.5	5	7
Class 3: 6.5 – 9.5	2	9
Class 4: 9.5 – 12.5	1	10

Note: the first column may start at 0.5 instead of 0; the histogram below starts its first column at 0

Histogram:



Class midpoint (x_m): take the average of the sum of the lower boundary and the upper boundary (or the average of the sum of the lower limit and the upper limit) (0.5+3.5)/2 = 2 (or (1+3)/2 = 2); (3.5+6.5)/2 = 5 (or (4+6)/2

= 5); (6.5+9.5)/2 = 8 (or (7+9)/2 = 8); (9.5+12.5)/2 = 11 (or (10+12)/2 = 11)

Class midpoints (x_m): 2, 5, 8, and 11

Note: it is preferable that the class-width be an odd number. This ensures that the midpoint of each class has the same place value as the data

Note: the data of the histogram is uni-modally distributed

Mean (μ): multiply the class midpoint with the corresponding class frequency and sum the results for all classes; then divide the result by the sum of the frequencies

Class midpoints times class frequencies: 2*2 = 4, 5*5 = 25, 8*2 = 16, and 11*1 = 11

Sum the results: 4+25+16+11 = 56

Summation of all frequencies: 2+5+2+1 = 10

Mean (μ): 56/10 = 5.6

Median (MD): observe the cumulative frequencies for the data (previous page) and note that with 10 numbers, the median numbers are the fifth and sixth number in the sorted data; the fifth and sixth numbers are in Class 2

Calculate the Median (MD) with the following formula: MD = $L + \left(\frac{\frac{n}{2} - CF}{f}\right) i$

L = lower limit of the class containing the median value(s)

n = cumulative frequency

CF = the cumulative frequency of the class(es) preceding the median class

f = the frequency of the class containing the median value(s)

i = class-width of the median class

Then:

L = 4

n = 10

CF = 2

f = 5

i = 3

MD = $4 + \left(\frac{\left(\frac{10}{2}\right) - 2}{5}\right) * 3 = 4 + \left(\frac{5-2}{5}\right) * 3 = 5.8$

Mode: the class(es) with the highest frequency, Class 2

Note: when raw data is grouped and statistics/parameters are calculated from the classes, accuracy gets lost.