

Question 1

How many square units are in the area of the triangle whose vertices are the x and y intercepts of the curve $y = (x + 2)(x - 3)^2$?



Solution 1

ANS: 45 Square units



If the area of the outer square is 1, what is the area of the small square inside?



Solution 2

ANS: Area of the small square is $\frac{1}{2}$.



Question 3

One third of the number 27^{81} can be written in the form 9^A where A is a three digit number. Find the value of A .

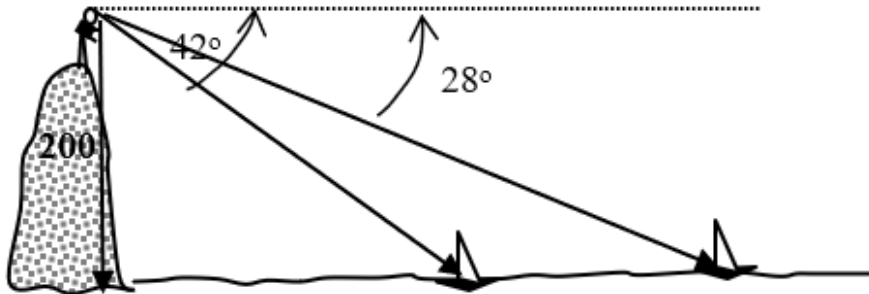


ANS: $A = 121$



Question 4

From the top of a cliff, 200 feet high, an observer measures the angles of depressions of two ships right in front of his sight to be 28° and 42° respectively. See the figure below.



What is the distance between the two ships?



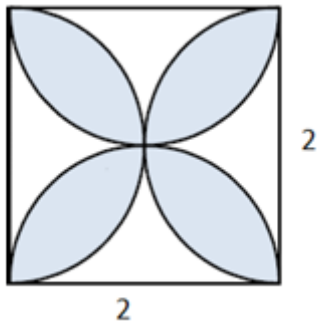
Solution 4

ANS: 154.02 feet



Question 5

The figure shows a square and four semicircles generated with each side of the square as a diameter. If the side length of the square is 2, find the area of the shaded region.



Solution 5

ANS: Area of the shaded region is
 $2\pi - 4$



Question 6

Amber, Ben, and Cathy shared a pizza. Amber ate $\frac{1}{5}$ of the pizza, Ben ate one-half as much as Cathy did. Find how much of the pizza Ben ate



Solution 6

ANS: $\frac{4}{15}$



Question 7

If you select at random an integer number from 66 to 201, both included, what is the probability that the number is divisible by 2 or 3?



Solution 7

$$\text{ANS: } \frac{91}{136} \simeq 0.67$$



Question 8

Let a and b be real numbers such that $a \neq b$, $a^2 + 3a = 2$, and $b^2 + 3b = 2$.
 Find the value of $(1 + a)(1 + b)$.



Solution 8

ANS: -4



Question 9

Let z be a complex number such that $z^5 = 1$ and $z \neq 1$ Compute

$$z + \frac{1}{z} + z^2 + \frac{1}{z^2}$$



Solution 9

ANS: -1



Question 10

Let $f(x) = (2 - \sin \sqrt{x})^2$

What is the maximum value of $f(x)$?



Solution 10

ANS: 9



Question T1

In order to buy a book, Emma needed 7 more cents but Kim needed 1 more cent. They decided to combine their money but even then they did not have enough money to buy the book. How much did the book cost?



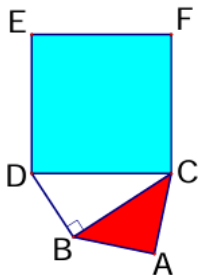
Solution T1

ANS: 7



Question T2

In the accompanying figure $\triangle ABC$ is an isosceles right triangle with the right angle at A and each leg's length equals to 28 cm. The triangle $\triangle DBC$ is a right triangle in which the leg BD is 21 cm. What is the area of the square $DEFC$?



Solution T2

ANS: 2009cm^2



Question T3

If a and b are the solutions of the equation $x^2 + 2019x - 2020 = 0$. What is the value of

$$\frac{1}{a} + \frac{1}{b}$$



Solution T3

$$\text{ANS: } \frac{-2019}{2020}$$



Question T4

In the series a_n , every three consecutive terms are the edges of a right triangle with the last term the hypotenuse. For instance, if the first two terms are 3, and 4, then the third term will be 5, because 3, 4, 5 are the three edges of a right triangle. Assume $a_1 = 3$, $a_2 = 4$. What is the length of the hypotenuse of the 10th right triangle according to the series?



Solution T4

$$\text{ANS: } \sqrt{1919} = 43.81$$



Question T5

Suppose z_1 , z_2 , and z_3 are the three roots of $z^3 - 18z - 8 = 0$. Simplify

$$A = \frac{(z_1 - z_2)^2(z_2 - z_3)^2(z_3 - z_1)}{200}$$



Solution T5

ANS: $A = 108$



Question T6

Find the value of x if

$$3^x - 2^x = \sqrt{6^x}$$



Solution T6

ANS:

$$x = \frac{2 \ln(1 + \sqrt{5}) - \ln 4}{\ln 3 - \ln 2}$$



Question T7

Suppose p and q are two distinct numbers and the $f(x) = x^2 + px + q$ satisfies $f(p) = f(q)$. What is the value of $f(2)$?



Solution T7

ANS: 4

